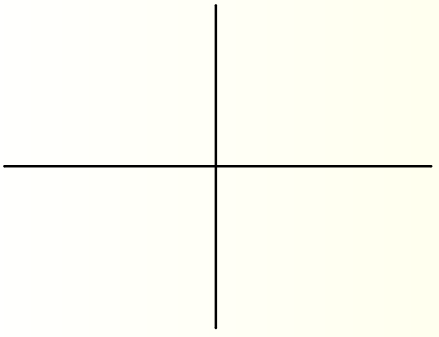
Calculus Warm Up #5-4

Sketch the solid created when the graph of

$y = x \cos x + 2$

is revolved about the x-axis on the interval [-2, 2]. (Calculator in radian mode. Fuss with your window to make an accurate sketch.)



Write the integral and calculate the volume.

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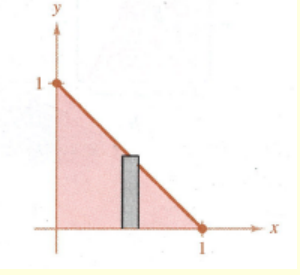
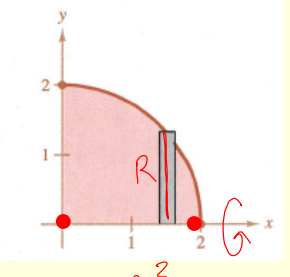
In Exercises 1–8, find the volume of the solid formed by revolving the given region about the x-axis.

1. $y = -x + 1$



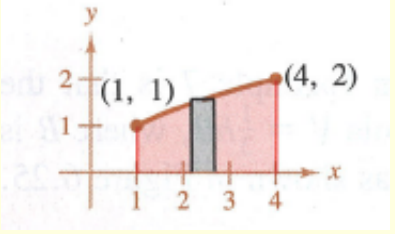
3. $y = \sqrt{4 - x^2}$



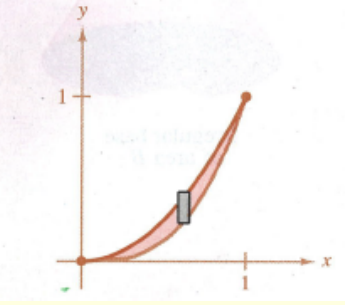
$$\pi \int_0^2 \left(\sqrt{4 - x^2}\right)^2 dx$$

In Exercises 1–8, find the volume of the solid formed by revolving the given region about the x-axis.

5. $y = \sqrt{x}$

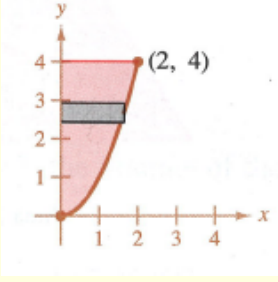


7. $y = x^2, y = x^3$

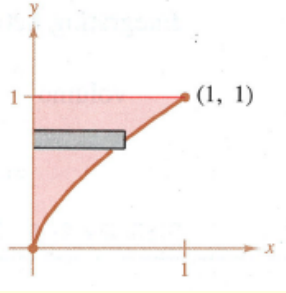


In Exercises 9–12, find the volume of the solid formed by revolving the given region about the y-axis.

9. $y = x^2$



11. $y = x^{2/3}$



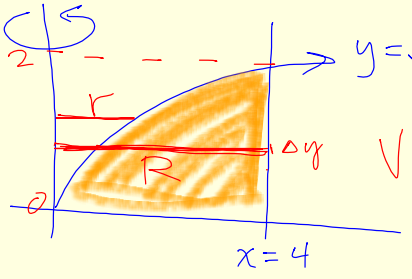
In Exercises 13–16, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the indicated lines.

13. $y = \sqrt{x}$, $y = 0$, $x = 4$

(a) the x-axis (b) the y-axis

(c) the line $x = 4$ (d) the line $x = 6$

b)



$y = \sqrt{x} \rightarrow x = y^2$

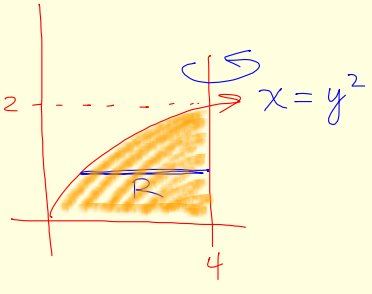
$$V = \pi \int_0^2 \left[(4^2) - (y^2)^2 \right] dy$$

In Exercises 13–16, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the indicated lines.

13. $y = \sqrt{x}$, $y = 0$, $x = 4$

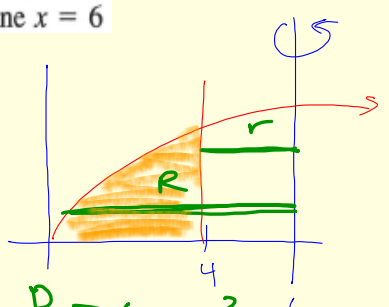
(a) the x-axis (b) the y-axis

(c) the line $x = 4$ (d) the line $x = 6$



$R = 4 - y^2$

$$V = \pi \int_0^2 (4 - y^2)^2 dy$$



$R = 6 - y^2$

$r = 2$

$$\pi \int_0^2 \left[(6 - y)^2 - 2^2 \right] dy$$

6.2 Solids of revolution

✓ The disc method

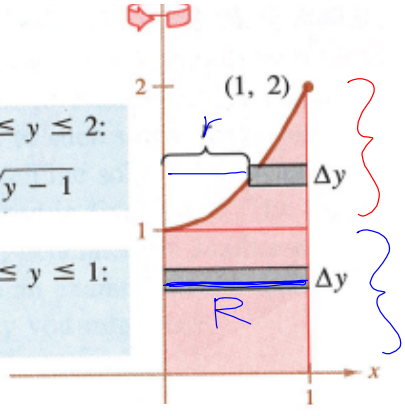
✓ The washer method

Solids with known cross sections

and extra credit project. :)

EXAMPLE 4 Two-integral case, integrating with respect to y

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis, as shown in Figure 6.20.



For $1 \le y \le 2$:
$R = 1$
$r = \sqrt{y - 1}$

For $0 \le y \le 1$:
$R = 1$
$r = 0$

$$V = \pi \int_1^2 \left[1^2 - \left(\sqrt{y-1}\right)^2\right] dy$$

$$V = \pi \int_0^1 1^2 \, dy$$

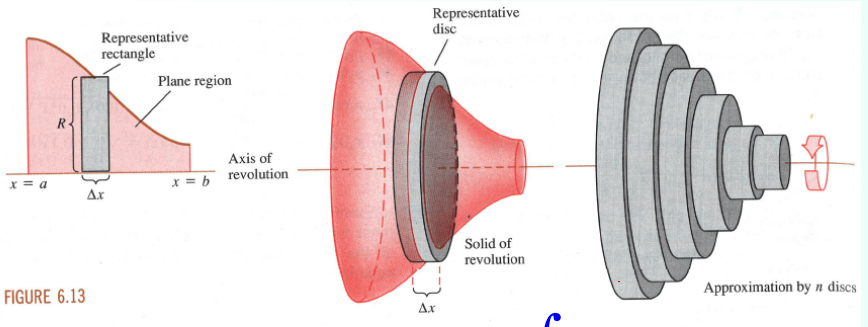


FIGURE 6.13

$$V = \pi \int [R(x)]^2 dx$$

Cross section: Circle

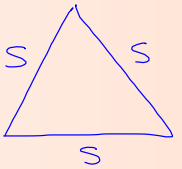
A(x) = area of the cross section

$$V(x) = \int A(x)\, dx$$

Solids with known cross sections:

Common cross sections:

Square, rectangle, triangle, semi circle, trapezoid



equilateral Δ

$$A = \frac{\sqrt{3}}{4} s^2$$

1. For cross sections of area $A(x)$, taken perpendicular to the x-axis,

$$\text{volume} = \int_a^b A(x)\, dx.$$

2. For cross sections of area $A(y)$, taken perpendicular to the y-axis,

$$\text{volume} = \int_c^d A(y)\, dy.$$

(See Figure 6.22.)

Find the volume of the solid whose base is the area bounded by the lines

$$f(x) = 1 - \frac{x}{2}, \qquad g(x) = -1 + \frac{x}{2}, \qquad \text{and} \qquad x = 0$$

and whose cross sections perpendicular to the x-axis are equilateral triangles.

$A = \frac{\sqrt{3}}{4} s^2$

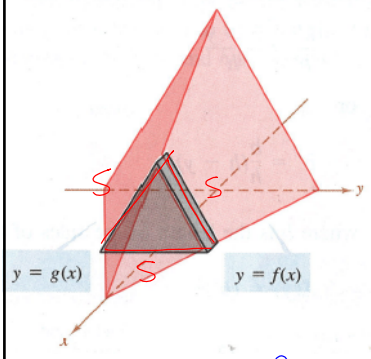
$s = f(x) - g(x)$

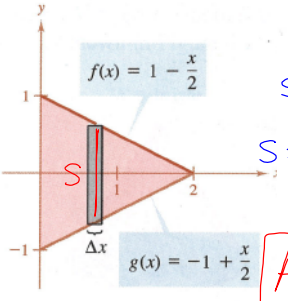
$s = 1 - \frac{x}{2} - \left(-1 + \frac{x}{2}\right)$

$s = 2 - x$

$A(x) = \frac{\sqrt{3}}{4}(2-x)^2$

area of a cross section





Triangular base in xy-plane

$V = \int A(x)\,dx$

$V = \frac{\sqrt{3}}{4}\int_0^2 (2-x)^2\,dx$

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15 - 27 odd, 45

Friday:	HW Quiz	Group Quiz
	pgs. 278, 287,	6.1 area,
	289, 299	6.2 volume discs.